E–0382

Sub. Code
1PMA1C1

M.Phil. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

RESEARCH METHODOLOGY

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A (5 × 5 = 25)

Answer any **five** questions.

1. Define data and give some examples of data. Distinguish between primary data and secondary data.

2. Define research and discuss the need for research.

3. Narrate the general format of a thesis in detail.

4. Define the following terms. Give an example for each.

   (a) R-module

   (b) Annihilator of an R-module

   (c) Faithful R-module.

5. Let M, N and P be R-modules. Prove the following :

   (a) $(M \otimes N) \otimes P \simeq M \otimes (N \otimes P)$

   (b) $M \otimes N \simeq N \otimes M$

6. With the usual notations, prove that R_s is a flat R – module.

7. Let R be a Noetherian ring and M a finitely generated R-module. Prove that $\bigcap_{P \in Ass(M)} \sqrt{Ann_R(M)}$.

8. If R is an Artinian ring, then prove that the Jacobson radical is nil potent.

Section B (5 × 10 = 50)

Answer **all** questions, choosing either (a) or (b).

9. (a) Enumerate the following terms :

   (i) Problems definition

   (ii) Research design

   (iii) Data collection

   (iv) Data analysis

   (v) Interpretation of results.

Or

(b) (i) What is a mail survey? What are the tasks in mailing questionnaire?

   (ii) Give brief account of editing primary data.

10. (a) Write a short notes on :

   (i) Footnotes

   (ii) Reference

   (iii) Appendix.

Or

(b) Narrate a short notes on tables and figures in a thesis.

11. (a) Show that the tensor product of M and N exists and it is unique up to isomorphism.

Or

(b) Let A be a flat R – algebra and M and N are R-modules. Let φ_M be the natural map. Prove the following :

(i) φ_M is injective if M if finitely generated

(ii) φ_M is an isomorphism if M is finitely presented.

12. (a) State and prove Chinese Remainder theorem.

Or

(b) (i) State and prove Nakayama lemma

(ii) Prove that R is a local ring if and only if it has a unique maximal ideal.

13. (a) State and prove Hilbert's basis theorem.

Or

(b) (i) State and prove First uniqueness theorem on primary decomposition.

(ii) State and prove second uniqueness theorem on primary decomposition.

E–0383

Sub. Code
1PMA2C1

M.Phil. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

ANALYSIS

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Section A $(5 \times 5 = 25)$

Answer any **five** questions.

1. Define the following terms :

   (a) Measurable space

   (b) Borel set

   (c) G_δ -set

   (d) F_δ -set

   (e) Simple function.

2. Let $\{E_K\}$ be a sequence of measurable sets in X, such that $\sum_{K=1}^{\infty} \mu(E_K) < \infty$. Show that almost all $x \in X$ line in at most finitely many of the sets E_K.

3. State and prove the Vitali-Caratheodory theorem.

4. Define the following terms:

   (a) L^p space

   (b) Convex function

   (c) Signed measures

   (d) Lebesgue decomposition.

5. Prove that the total variation $|\mu|$ of a complex measure μ on $\mathfrak{M}$ is a positive measures on $\mathfrak{M}$.

6. Define product measure. If $E \in S \times J$ then prove that $E_x \in J$ and $E^y \in S$ for every $x \in X$ and $y \in Y$.

7. For every $x \in A$, prove that $\sigma(x)$ is compact and not empty.

8. Define quotient norm. Show that x is invertible if and only if $h(x) \neq 0$ for every $h \in \Delta$.

Section B (5 × 10 = 50)

Answer **all** questions, choosing either (a) or (b).

9. (a) State and prove Fatou's lemma. Show by an example that the inequality in the lemma may be strict. Also prove the Lebesgue's dominated convergence theorem.

Or

(b) (i) Suppose $f : X \to [0, \infty]$ is measurable , $E \in \mathfrak{M}$ and $\int_E fd\,\mu = 0$. Prove that $f = 0$ almost every where on *E*.

(ii) Suppose $f \in L'(\mu)$ and $\int_E f\,d\mu = 0$ for every $E \in \mathfrak{M}$. Shows that $f = 0$ almost everywhere on *X*.

(iii) Suppose $f \in L'(\mu)$ and $\left|\int_X fd\,\mu\right| = \int_X |f|\,d\mu$. Prove that there is a constant α such that $\alpha f = |f|$ almost everywhere on *X*.

10. (a) (i) State the Riesz representation theorem.

(ii) If $A \subset R'$ and every subset of A is Lebesgue measurable then prove that $m(A) = 0$. Also prove that every set of positive measures has non measurable subsets.

Or

(b) (i) Let *X* be a locally compact Hausdorff space in which every open set is σ-compact. Let λ be any positive Borel measure on *X* such that $\lambda(k) < \infty$ for every compact set k. Prove that λ is regular.

(ii) State and prove Lusin's theorem.

11. (a) (i) State and prove the Minkowski's inequality.

(ii) Can you say $C_c(X) \subset C_o(X)$? Why?

Or

(b) State and prove the Lebesque-Radon-Nikodym theorem.

12. (a) Let (X,S,μ) and (Y,J,λ) be $\sigma-$ finite measure space. Suppose $Q \in S \times J$. If $\varphi(x) = \lambda(Q_x)$, $\psi(y) = \mu(Q^y)$ for every $x \in X$ and $y \in Y$, then prove that φ is S-measurable, ψ is J-measurable and

$$\int_X \varphi \, d\mu = \int_Y \psi \, d\lambda .$$

Or

(b) State and prove the Fubinis theorem.

13. (a) (i) What is meant by Bonach algebra? Give an example.

(ii) State the spectral radius formula.

(iii) State and prove Gelfand-Mazur theorem.

Or

(b) Suppose $f(e^{i\theta}) = \sum_{-\infty}^{\infty} C_n \, e^{in\theta}$, $\sum_{-\infty}^{\infty} |C_n| < \infty$ and $f(e^{i\theta}) \neq 0$ for every real θ. Prove that $\frac{1}{f(e^{i\theta})} = \sum_{-\infty}^{\infty} \gamma_x e^{in\theta}$ with $\sum_{-\infty}^{\infty} |\gamma_x| < \infty$.